Print This Page

Building Bridges with Similar Triangles

Standards

Primary

CCSS.Math.Content.HSG-SRT.A.3Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

CCSS.Math.Content.HSG-SRT.B.5Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

CCSS.Math.Content.HSG-SRT.C.6Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

CCSS.Math.Content.HSG-SRT.C.7Explain and use the relationship between the sine and cosine of complementary angles.

CCSS.Math.Content.HSG-SRT.C.8Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Building Bridges with Similar Triangles

Lesson Overview

Lesson Summary

Students examine and explore the concept of similar triangles by applying knowledge about similar figures and transformations. Students learn about the trigonometric ratios and use them to identify similar triangles. They explore patterns in the relationship between the sine and cosine of complementary angles. Finally, students apply their knowledge of similar triangles, the Pythagorean Theorem, and trigonometric ratios to help two competitors complete an obstacle course.

Essential Questions

- How are triangles used in bridge building?
- How can I identify similar triangles?
- How can geometric principles be applied to your life?
- How can I show that two triangles are similar using AA?
- How can I use congruence to show that two triangles are similar?

- How are sine and cosine related?
- How can I use the Pythagorean Theorem to identify right triangles?
- How can I use trigonometry to finish an obstacle course?

Sessions

- **Session 1:** Identifying Similar Triangles Using Angles
- **Session 2:** Introducing the Trigonometric Ratios
- **Session 3:** Trigonometry in Action

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them
- 3. Construct viable arguments and critique the reasoning of others
- 4. Model with mathematics
- 5. Use appropriate tools strategically
- 6. Attend to precision
- 8. Look for and express regularity in repeated reasoning

Building Bridges with Similar Triangles

Session 1: Identifying Similar Triangles Using Angles

Session Summary

Students view a video, which introduces how triangles are used to build bridges. The students use what they learn from the video to discuss real-life applications of triangles and to develop the AA postulate. They learn why triangles are useful in bridge building and how engineers apply geometric principles.

CCSS Math Standards

- **HSG-SRT.A.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- **HSG-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them
- 3. Construct viable arguments and critique the reasoning of others
- 4. Model with mathematics
- 6. Attend to precision
- 8. Look for and express regularity in repeated reasoning

Essential Questions

- How are triangles used in bridge building?
- How can I identify similar triangles?
- How can geometric principles be applied to your life?

Task

- **Identifying Similar Triangles:** Students apply the angle sum theorem of triangles to determine if triangles are similar. **(HSG-SRT.A.3, HSG-SRT.B.5)**

Teacher Preparation

- **“Video Discussion Guide: Triangles: Bridges of Support”** (one copy for each student)
- **“Identifying Similar Triangles”** (one copy for each student)
- **“How Did You Work: Bridges of Support”** (one copy for each student)
- “Math and the Greeks”
- **Triangles: Bridges of Support** [4:22]

Instructional Procedure Chart

Section	Time	Instructional Sequence
Activate Thinking: Similar Shapes	5 minutes	On the board, draw a group of similar triangles, a group of similar rectangles, a group of similar irregular pentagons, and a group of similar concave hexagons and label each as group a, b, c, and d, respectively. Assess prior knowledge of similar shapes. **Say:** *In your notebooks, write down anything you notice about these groups of shapes on the board.* **Ask:** *What do you notice about group a? Group b? Group c? Group d?* (Sample responses: They’re the same, but different sizes; they have the same shape; they have the same angles.) **Ask:** *What do we call shapes that are almost the same, but have different sizes?* (Sample responses: similar, proportional) **Say:** *When shapes look the same, but are different sizes, we call them similar.*

Ask: *What can you say about the angles of the similar shapes?* (Sample response: The angles of similar shapes are equal or congruent.)

Ask: *Can anyone give me some examples of where triangles are used in real life?* (Sample responses: buildings, tiles, pictures, clothes, design)

Say: *Today you are going to learn about triangles and how they are used in building bridges. Keep an eye out for similar triangles.*

Background Information: Similar shapes are shapes whose corresponding angles are congruent. The figures are the same shape, but different sizes.

Media Analysis: Triangles: Bridges of Support — 13 minutes

Task: Identifying Similar Triangles

Give each student a copy of **"Video Discussion Guide: Triangles: Bridges of Support."**

Say: *We are now going to see how triangles are used to build bridges. As we view the video, I will stop to ask some questions, so you should be paying very close attention and use "Video Discussion (Triangles: Bridges of Support)" to help you take notes. This will act as a guide for you over the next couple of lessons. If there is something you don't understand or want to know more about, write it down and we will discuss it at the end. You should also be working sample problems from the video as we pause it.*

Show students the first part of the video Triangles: Bridges of Support [4:22].

Pause the video at 1:24.

Ask: *What do you notice about those triangles?* (Sample responses: They're different sizes; they are similar; they have the same shape.) Wait until you get the response that they are similar. Don’t worry about proving similarity; just bring up the concept again.

Resume playing the video. Pause the video at 1:38.

Ask: *Do you see any similar triangles? Where are they?* (Sample response: yes/no) If the students don’t see them right away, point some out. There are some similar triangles visible. They are the small right triangle with the arrows and the larger right triangle to the left. Also, there are the isosceles triangles with 2 sets of arrows and the larger one that is to the left.

Ask: *What do you know about the three angles of a triangle?* (Sample response: They add to 180 degrees.)

Resume playing the video. Pause the video at 2:17.

Ask: *What are the four types of triangles?* (equilateral, right, scalene, and isosceles)

Resume playing the video. Pause the video at 2:31.

Ask: *What is the area of that triangle?* (12 square feet)

Resume playing the video. Pause the video at 2:47.

Ask: *What is the length of ab?* (4 ft)

Resume playing the video. Pause the video at 3:45.

Ask: *What is this relationship called?* (Sample responses: the Pythagorean Theorem, $a^2 + b^2 = c^2$)

Resume playing the video. Pause the video at 4:10.

Ask: *What is the length of b?* (36 ft)

		Resume playing the video. Stop the video at 4:21. **Say:** *Check your work by comparing it to what is on the screen. Raise your hand if you have any questions.*
Build Background: Post-Video Discussion	5 minutes	Have students form groups of 4. **Say:** *You are now going to share the answers you wrote on the activity sheet with the students in your group to ensure that everyone has a good understanding of what was covered in the video. Raise your hands when you have finished discussing your answers to receive the next assignment.* When groups are done, distribute a copy of **"Identifying Similar Triangles"** to each student.
Investigating Mathematics: Identifying Similar Triangles	10 minutes	Have students work in small groups to complete **"Identifying Similar Triangles."** As students are working on their activity page, monitor their strategies: • How do students approach the task? • How do students determine missing angles? • Do students realize that they can use 2 angles to identify similar triangles? **Ask:** *Why is it that type of triangle?* (Sample responses: It has a right angle, it has 3 equal sides/angles; it has no equal sides/angles.) **Ask:** *What do you notice about triangles with the same angle measures?* (Sample responses: They're the same shape; they're similar.)

Conduct a whole-class discussion to review **"Video Discussion Guide: Triangles: Bridges of Support"** and **"Identifying Similar Triangles."**

Ask for responses from the discussion guide:

1. What are reasons that triangles are useful in bridge building? (Sample responses: Triangles are rigid; triangles can hold a lot of weight; triangles cannot collapse or change shape unless one of their sides is changed; triangles can split the original support onto the two sides of the triangle.)
2. What are the three types of triangles? (equilateral, isosceles, and scalene)
3. What is the formula for finding the area of a triangle? ($A = \frac{1}{2}bh$)
4. What is the "special relationship" between a right triangle's legs? (The Pythagorean Theorem: the sum of the squares of the legs is equal to the square of the hypotenuse.)

Communicating Mathematical Ideas: Triangles	7 minutes	Make sure students correct their work.

Ask: *What do you notice about similar triangles?* (Sample response: They have the same angle and shape.)

Ask: *How can you identify similar triangles?* (Sample response: They have two pairs of congruent angles.)

Ask: *Do you need to know the measure of all three angles? Why?* (Sample response: No, you need only two, because you know that if there are two pairs of equal angles, the third pair will be equal.)

Call on groups to share their responses.

Ask: *What other ways are triangles used in real life?* (Sample responses: building houses: roof height measurements, fashion design: in order to create patterns that have similar lines and shapes)

Ask: *What other occupations might require the application of geometric knowledge?* (Sample responses: building, construction, design, art)

Ask: *How can you apply geometric principles in your life?* (Sample responses: building a dog or bird house, designing an outfit, using a skateboard ramp)

Ask: *What might happen if bridge engineers used other shapes to build bridges?* (Sample responses: The bridge would collapse; it wouldn't be as strong.)

Once students have had a chance to offer some possibilities for applications of triangles in the real world, distribute **"How Did You Work: Bridges of Support."**

Say: *You will have 2 minutes to complete this activity page.*

Ask: *What can we say about triangles that have two equal angles?* (Sample response: They are similar triangles.)

Ask: *Is there anything that was covered in the video that you have questions about?* (yes/no) At this point, everything in the video should have been discussed, but there may be a question about suspension bridges.

Wrap-Up	5 minutes	

Say: *To learn more about the Pythagorean Theorem, you can play Math and the Greeks. This is a game where you help Pythagoras make his way out of the temp maze by proving the Pythagorean Theorem. Study the properties of triangles and Euclid's Proof of the Pythagorean Theorem in order to escape the maze and avoid being devoured by the Minotaur.As you play, take notes on the different ways to prove the Pythagorean Theorem and how it is used to find the lengths of the sides of triangles.*

You may decide if you want this homework activity to be mandatory or extra credit.

- Bold text in the Instructional Sequence highlights explicit instruction to the Common Core standards.

Building Bridges with Similar Triangles

Session 2: Introducing the Trigonometric Ratios

Session Summary

Students discuss similarity and then view a video that introduces the trigonometric ratios. Once they know the trigonometric ratios, students develop an understanding of how to determine similar right triangles and the relationship between sine and cosine of complementary angles.

CCSS Math Standards

- **HSG-SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **HSG-SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them
- 3. Construct viable arguments and critique the reasoning of others
- 5. Use appropriate tools strategically
- 6. Attend to precision
- 8. Look for and express regularity in repeated reasoning

Essential Questions

- How can I show triangles are similar other than using two angles?
- What is the relationship between sine and cosine?

Task

- **Finding Ratios in Right Triangles:** Students calculate the values of the trigonometric ratios based on given information. From these calculations, they learn the relationship between sine and cosine of complementary angles. **(HSG.SRT.C.6, HSG-SRT.C.7)**

Teacher Preparation

- Graphing calculators
- **“Finding Ratios in Right Triangles”** (one copy for each student)

- [Introduction: Triangles and Trigonometry] [1:01]
- [“Math and the Greeks”]

Instructional Procedure Chart

Section	Time	Instructional Sequence
Activate Thinking: Proportionality	7 minutes	Begin the session by telling students that today they are going to learn about another way to identify similar triangles. Assess prior knowledge of identifying similar triangles and using the Pythagorean Theorem. **Ask:** *Yesterday we developed a method of identifying similar triangles. Please take out your notes and tell me what it was*. (Sample response: If two angles of a triangle are the same, the triangles will be similar.) **Ask:** *Why was that true?* (Sample response: Similar shapes have equal corresponding angles.) **Ask:** *How could we say two triangles are similar if we know only two angles?* (Sample response: The three angles of any triangle add to 180 degrees, and if we know two pairs of angles are equal, the third angle pair must be equal too.) **Say:** *Today we will be using another method of identifying similar triangles.* **Ask:** *What can we say about all similar shapes?* (Sample responses: They have equal/congruent angles; they’re the same shape; they’re different sizes.) If students are stumped, draw some similar shapes to remind them. **Ask:** *What is a different way of describing objects that are the same shape, but different sizes?* (Sample responses: They’re similar; they’re scaled; they’re proportional.) Here you want to bring up the idea of a scale factor and proportionality. It might help to ask students how they described scale models of maps.

Say: *So similar shapes have equal angles and are proportional. You should add that to your notes.* You may need to write that on the board.

Ask: *Does anyone remember another time we used proportionality?* (Sample responses: algebraic growth, ratios, proportions) If they're not coming up with proportions or ratios, tell them to focus on the basis of the word proportionality.

Ask: *What are ratios in general?* (Sample responses: fractions, a comparison of two quantities, decimals)

Write some proportional ratios on the board, but don't write the = sign. Some examples are $\frac{3}{4}$ & $\frac{9}{12}$, $\frac{6}{9}$ & $\frac{18}{27}$, $\frac{4}{5}$ & $\frac{16}{20}$.

Ask: *Here are some proportional ratios. What can you say about them?* (They are equal.) *Can you prove that?* (Sample response: Yes, if you divide the numerator by the denominator, you get equal amounts.)

Ask: *So what can we say about proportional ratios?* (They are equal.)

Say: *Today we are going to be using the idea that proportional ratios are equal to show that triangles are similar.*

Background Information: Students should be familiar with similar shapes and recognize that similar figures have the same shape and congruent angles. Based on this information and the work from the previous session, students should realize that when working with triangles, they only need to know the measures of two sets of angle pairs to identify similar triangles. The AA similarity is based on the angle sum theorem of triangles.

		Task: Finding Ratios in Right Triangles
Media Analysis: Introduction: Triangles and Trigonometry	10 minutes	**Say:** *We are now going to learn about the special ratios of right triangles. As we view the video, I will stop to ask some questions, so you should be paying very close attention and taking notes in your notebooks.If there is something you don't understand or*

want to know more about, write it down and we will discuss it at the end.

Show students the video **Introduction: Triangles and Trigonometry** [1:01].

Pause the video at 0:33.

Ask: *What is the Pythagorean Theorem?* (Sample responses: sum of the squares of the legs equals the square of hypotenuse, $a^2+b^2=c^2$)

Resume playing the video. Pause the video at 0:47.

Ask: *Has anyone heard of trigonometry and know what it is?* (Sample responses: more math, it deals with angles)

Say: *Trigonometry is a branch of math that studies right triangles and the relationship between the lengths of the sides and the measures of the angles. There are three primary ratios in trigonometry: sine, cosine, and tangent. We will learn about all three and apply them to real-life situations. Copy this in your notes.*

Resume playing the video. Pause the video at 1:02. Have students copy what is on the screen.

Ask: *What are the ratios for a right triangle?* (Sample response: sine, cosine, tangent, a over c, b over c, a over b)

Ask: *Earlier, we determined that ratios are a comparison of two quantities. What are the right triangles comparing?* (Sample responses: the sides, the angles, the angles and sides) If they don’t agree on them being a comparison of the lengths of the sides, ask what the lower and upper case letters represent and where they are in the equations.

Ask: *What angle is being focused on in this example?* (Angle A)

Say: *The tricky thing about trigonometry is that the ratios are dependent on the angle. That angle is usually called theta, but it can also be named by other variables. Seeing theta does not necessarily mean that two angles are congruent if they are in different problems. This is similar to how x is often used but changes from one problem to the next.* Draw theta and write the name, then tell students to copy it. *This is the most common variable used to indicate the angle measurement that you are using in a problem.*

Say: *Usually, instead of referring to the sides as a, b, and c, they are referred to as adjacent, opposite, and hypotenuse. These are all in relation to the angle theta.*

Draw a right triangle similar to the one on the screen and write theta where angle A is located.

Ask: *Can anybody guess which side is the hypotenuse?* (Sample responses: to the left, the longest side, c, AB) If they're stuck, ask them what they know about the hypotenuse. Then label that side.

Ask: *Can anybody guess which side is the opposite?* (Sample responses: to the right, the leg, a, BC, the medium side) If they're stuck, ask them to think what opposite means. Then label that side. Explain that the opposite side is the side that is intersected by both rays that make up angle theta.

Ask: *Can anybody guess which side is the adjacent?* (Sample responses: to the bottom, the shortest side, b, AC) If they're stuck, ask them to think of what adjacent means. Then label that side. Explain that the adjacent side is the ray of angle theta that is shorter than the other one.

Ask: *How could we rewrite the ratios using these side names?* (Sample responses: opposite over hypotenuse, adjacent over hypotenuse, opposite over adjacent) Write the more common form of the ratios and tell students to copy them and the picture into their notes.

Finish showing the video.

Build Background: Finding Ratios in Right Triangles	3 minutes	Distribute a copy of **“Finding Ratios in Right Triangles”** to each student. **Ask:** *How could you rewrite the ratios using just the first letter of each word?* (S=o/h C=a/h T=o/a) After that is rewritten, say each part as a word so the students learn the mnemonic device to remember the trigonometric ratios. **Ask:** *What is the shortcut to remembering the trigonometric ratios?* (Soh Cah Toa) **Ask:** *What did the mother tell her son to do when he stubbed his toe?* (Soh Cah Toa) **Say**: *You will now have a chance to practice comparing the sides of right triangles. In order to complete this assignment, you will have to use your calculator. To find the value of theta, follow these steps:* 1. *Calculate your ratio by dividing the sides.* 2. *Press 2^{nd}.* 3. *Press the trigonometric ratio button you want (sin, cos, tan).* Copy these steps onto the board.
Investigate Mathematics: Finding Ratios in Right Triangles	15 minutes	Have students work in small groups to complete **“Finding Ratios in Right Triangles.”** As students are working on their activity page, monitor their strategies: • How do students approach the task? • How do students determine which sides are adjacent, opposite, and hypotenuse? • Do students use the numerator/denominator for each ratio? **Ask:** *What did you identify as the opposite side? The adjacent side? The hypotenuse?* (Sample responses: It’s across from theta; it’s next to theta; it’s across from the right angle.)

Ask: *Can you figure out what the other angle measure will be? How?* (Sample response: Yes, use the angle sum theorem.)

Ask: *What do you notice about 2 and 3? 5 and 6? 7 and 8?* (Sample response: They're the same triangles.)

Conduct a whole-class discussion to review **"Finding Ratios in Right Triangles."**

Copy the problems onto the board.

Say: *Choose someone from your group to come up to the board and write your answers. Write your initials next to your work.*

Make sure students make corrections as the class reviews the answers.

Ask: *Do you agree or disagree with the work?* (yes/no)

Communicate Mathematical Ideas: Finding Ratios in Right Triangles	5 minutes	Make sure students fully explain how they did their work. If they are not going in depth, ask these questions: *What side is the adjacent? What side is the opposite? What side is the hypotenuse? How did you know which number goes where? How did you find the value of the unknown angle?*

Ask: *What is the relationship between sine and cosine?* (Sample response: They're the same when theta switches between the complementary angles; they're equal when the angles are complementary.)

Write the observations and summarize with a more formal summary of the relationship between the sine and cosine of complementary angles.

Ask: *How could you use the trigonometric ratios to identify similar triangles?* (Sample response: When the ratios were equal the angles were equal, so the triangles are similar.)

		Ask: *How does this work?* (Sample response: When the values of the ratios are the same, the same angle measure is being used. Using this and the AA rule of similarity, the triangles are similar.)
		Review the trigonometric ratios by asking students to name them and give the ratios. Write Soh Cah Toa on the board and fill in what each letter represents.
		Ask: *Is there anything that was covered in the video that you have questions about?* (yes/no) At this point, everything in the video should have been discussed.
		Ask: *What did you learn from your the prior session?* (Sample responses: There are different ways to prove the Pythagorean Theorem; how to use the Pythagorean Theorem.)
Wrap-Up	5 minutes	**Ask:** *What were the different ways to prove the Pythagorean Theorem?* (Sample responses: using the smaller squares that fit into the largest square, using 4 right triangles to cover a space that equals the squares of the sides, breaking up a big square to make the smaller squares)
		Ask: *What else did you learn?* (Sample responses: trigonometric ratios, how to use the Pythagorean Theorem)
		Say: *Now that you have some experience using the trigonometric ratios, you can play the Math and Greeks game again and see if you do any better. Is there anyone who needs another slip with the website on it?*

- Bold text in the Instructional Sequence highlights explicit instruction to the Common Core standards.

Building Bridges with Similar Triangles

Session 3: Trigonometry in Action

Session Summary

Students apply the trigonometric ratios to solve real-life problems. They also use the Pythagorean Theorem in applied problems related to two contestants trying to complete an obstacle course by using right triangles to cross different obstacles.

CCSS Math Standards

- **HSG-SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **HSG-SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **HSG-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them
- 4. Model with mathematics
- 5. Use appropriate tools strategically
- 6. Attend to precision

Essential Questions

- How can I use the Pythagorean Theorem to identify right triangles?
- How can I use trigonometry to finish an obstacle course?

Task

- **Obstacle Course:** Students apply their knowledge of right triangles to help Joan and Terry finish an obstacle course. **(HSG-SRT.C.6, HSG-SRT.C.7, HSG-SRT.C.8)**

Teacher Preparation

- **"Numbers for Finding Your Triangle"** (one copy of cut flash cards, one card for each student)
- Graphing calculators
- Using Trigonometric Functions in a Triangle [1:43]
- **"Obstacle Course"** (one copy for each student)
- Example 1: Cosine – Volcanoes [2:07]
- Example 2: Tangent – River Crossing [2:27]

Instructional Procedure Chart

Section	Time	Instructional Sequence
		Begin the session by telling students that today they are going to learn about using the trigonometric ratios to complete an obstacle course.
		Assess prior knowledge of trigonometric ratios and the Pythagorean Theorem. As you ask questions, write down key answers, such as the trigonometric ratios.
		Ask: *What are the trigonometric ratios we learned about yesterday?* (Sample responses: the ones for the right triangle, soh cah toa, sine, cosine, tangent)
		Ask: *What were they comparing?* (Sample responses: angles, sides, sides and angles)
Activate Thinking: Reviewing Trigonometric Ratios	5 minutes	**Ask:** *What are the trigonometric ratios?* (Sample responses: S=o/h C=a/h T=o/a, soh cah toa, sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, tangent equals opposite over adjacent, or similar using a, b, and c) Emphasize referring to the ratios in terms of opposite, adjacent, and hypotenuse; it will make them easier to remember.
		Ask: *What do you mean by opposite and adjacent?* (Sample responses: across from the angle, next to theta, the legs) Draw a right triangle and ask students to label the sides, but don't have theta there. Have them tell you they need theta and that the side names are relational to where theta is located. You can do this by asking how they know it is opposite/adjacent. It is opposite/adjacent to what? How do they know that angle?
		By the end, students should have directed the teacher in labeling a right triangle with theta and the appropriate side names. The trigonometric ratios should also be on the board.
		Say: *Make sure you have this in your notes from yesterday. If not, copy it now. You should have notes on the definitions of opposite and adjacent from yesterday, but if you do not, make sure you write them now. The opposite side is the side that is*

intersected by both rays that make up angle theta. The adjacent side is the ray of angle theta that is shorter than the other one.

Ask: *What types of triangles do they work with?* (Sample response: They work only with right triangles.) Students may not realize this. If not, ask them what trigonometry is. They should say it is the study of the sides and angles of right triangles.

Ask: *How do you know if you have a right triangle?* (Sample responses: It looks like it has an L in it; it has a right angle; the Pythagorean Theorem works.) The last one may not come from students. If that is the case, draw a 3, 4, 5 right triangle without the angle measures and only side lengths.

Ask: *Is this a right triangle? How do you know?* (Yes; the Pythagorean Theorem works.)

Say: *We are now going to play a little game to review the Pythagorean Theorem.*

Background Information: At this point, students should be familiar with the trigonometric ratios and be able to use them. They should also be able to identify right triangles using the Pythagorean Theorem and similar triangles using the AA theorem.

Build Background: Making a Right Triangle	5 Minutes	Pass out one of the cards from **"Numbers for Finding Your Triangle"** to each student. **Say:** *On the card you are receiving is a number that represents the length of the side of a right triangle. You must find the other two people whose number represents the length of a side of your triangle. You will have about 2 minutes to do this. Do not turn the number over until I tell you to.* Write the instructions on the board. Adjust the time depending on the skill of your students. You may want to give struggling students easier numbers (whole numbers that form Pythagorean triples). Pass out a number to each student. **Say:** *Okay, turn over your number and find your triangle.*

Ask: *How do you know those people belong in your group?* (Their numbers make the Pythagorean Theorem work with my number.)

Once students have found their groups, **Ask:** *How can we use this formula to solve right triangles?* (You can use the sum of the two legs to find the length of a hypotenuse.)

Play the game 2-3 times depending on your students' skill and behavior.

Say: *Now that we have practiced identifying right triangles, we will see how they can be used to cross a river. As we view the video, I will stop to ask some questions, so you should be paying very close attention and take notes in your notebook.If there is something you don't understand or want to know more about, write it down and we will discuss it at the end.*

Show students the video **Using Trigonometric Functions in a Triangle** [1:43].

Pause the video at :34.

Media Analysis: Using Trigonometric Functions in a Triangle

10 minutes

Say: *Copy the picture on the screen; be sure to label what the sides are in the picture. In place of the* $\frac{\pi}{4}$ *> write 45 degrees. Writing in degrees instead of in pi is just using a different unit of measurement – sort of like changing yards into meters.*

Ask: *How do you think we can use this information?* (Sample responses: Use trig ratios; use the Pythagorean Theorem.)

Say: *Try to solve this problem on your own.*

Give the class 2-3 minutes to work on this. Walk around seeing what students are doing. Don't say anything; just observe how many are able to set up the trigonometric ratios, fill in the known information, and solve for the unknown. Ask for volunteers to show what they did on the board; try to get a couple different methods and ask for explanations. Start a discussion on what other students think will work by asking if they agree with the reasoning of the volunteer and if that is what they were doing.

Say: *Let's see what they did in the video. Be sure to copy what the video does.*

Pause the video at 0:40.

Say: *So far so good?* This is a rhetorical question as you do a quick comparison to the work on the board. Note that the $\frac{\pi}{4}$ is replaced by 45 degrees.

Resume playing the video. Pause the video at :50.

Say: *The sine of 45° is equal to $\frac{\sqrt{2}}{2}$ which is equal to 0.7071. How does the work on the board compare so far?* Don't get into a long conversation about it; just get a sense of whether students think one method is working.

Resume playing the video. Pause the video at 1:10.

Ask: *How did the volunteers do?* (Sample responses: They were correct; it was okay, but there were some problems; they had the right idea.)

Review the work from the video and what is on the board.

Ask: *How did you know which ratios to use?* (Sample responses: I guessed; I had the hypotenuse so it couldn't be tangent; I have the opposite so it had to be sine.)

Ask: *How did you find the value of sin 45?* (Sample responses: I used a calculator; I used the charts in the back of the book.)

It may be necessary to stop and show students how to find the trigonometric ratios. Point out the trigonometric keys on the calculator. Tell them to press SIN and type 45 and press ENTER. This should give them Sin 45, which is about 0.7071.

To go in reverse, press 2nd, press SIN, type the ratio 0.7071, and press ENTER. They should get close to 45, the difference being rounding. As you are doing this demonstration, use an on-screen calculator or write the instructions on the board.

Finish reviewing the problem on the board by going over the steps of solving for *x.*

Finish showing the video.

Say: *That's interesting about truss bridges. Did you notice all the similar triangles used to build them? We're going to explore more of how to use trigonometric ratios to build bridges. Do you feel ready or do you want another example?*

If you feel they need more examples, use these videos: **Example 1: Cosine – Volcanoes** [2:07] or **Example 2: Tangent – River Crossing** [2:27]. If you decide to use either of these videos, pause them before each problem is worked out and ask students to try it on their own, suggest they use pictures. Then ask for volunteers to share their work. Ask these questions: *How did you label the sides? Which is the opposite/adjacent/hypotenuse? How did you know which ratio to use? What steps did you take to solve for x?*

Task: Obstacle Course

Distribute a copy of **"Obstacle Course"** to each student.

Build Background: Obstacle Course	3 minutes	**Ask:** *What are the trigonometric ratios?* (Sample responses: S=o/h C=a/h T=o/a, soh cah toa, sin equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, tangent equals opposite over adjacent, or similar using a, b, and c) Try to emphasize thinking of the ratios in terms of opposite, adjacent, and hypotenuse; it will make them easier to remember.
		Ask: *How can we use these?* (Sample responses: to find missing sides, to find missing angles, to cross a river)
Group Application:	12 minutes	**Say:** *You are now going to apply what you have learned about trigonometric ratios to help two students complete an obstacle*

course. You will have about 6-9 minutes to work on this. What you don't finish in class will be your homework for tonight.

The time differences noted in this and the following sections are based on whether you showed an extra video in the Media Analysis section.

Have students work in small groups to complete **"Obstacle Course."**

Obstacle Course		As students are working on their activity page, monitor their strategies: • How do students approach the task? • How do students determine which ratios to use? • Do students label the sides of the triangles correctly?

Ask: *Why did you choose that ratio?* (Sample responses: It's the one that fits with the sides I have; I need to find the hypotenuse/opposite/adjacent side.)

Ask: *How did you solve for x?* (Sample responses: inverse operations; I set up the equation)

Conduct a whole-class discussion to review **"Obstacle Course."**

Have students come to the board to share and explain their work.

Communicating Mathematical Ideas: Obstacle Course	5minutes	Make sure students correct their work and that the other students are asking questions.

Ask: *Do you agree with the work on the board? Was their approach what you did?* (Sample responses: yes/no, they used the wrong ratio, I did it a different way) Encourage students to compare different methods of solving these problems by asking how they did it differently.

Ask: *Which ratio did you use? Why?* (Sample responses: sine, cosine, tangent, because it fits with the information I have)

Review how trigonometric ratios can be used to build bridges to cross a river.

Ask: *How did we use the trigonometric ratios to build bridges?* (Sample responses: by solving for *x* or theta, by multiplying the sides by the ratio, by using Pythagorean Theorem)

Ask: *Is there anything that was covered in the video that you have questions about?* (yes/no) At this point, everything in the video should have been discussed.

Ask: *What have we been studying the last few days?* (Sample responses: triangles, similar shapes, trig ratios, soh cah toa)

Ask: *What do all these things relate to?* (triangles)

Wrap-Up	5 minutes	**Ask:** *What are the different types of triangles and how do we identify them?* (Sample responses: equilateral, right, scalene, isosceles, by their sides and angles)

Ask: *How do we identify similar triangles?* (by their angles OR proportional side lengths)

Ask: *Why do we only need to know the measures of two angle pairs?* (Based on the angle sum theorem of triangles, we know that if there are two pairs of equal angles, the third pair must be equal because there is only one value that can be added to the other two to make 180 degrees.)

Ask: *What was the special type of triangle we focused on for the last two days?* (right triangle)

Ask: *How can you identify right triangles?* (Sample responses: Look for the right angle; use the Pythagorean Theorem.)

Ask: *What did you learn about it?* (Sample responses: There are some trigonometric ratios; soh cah toa.)

Ask: *What are the trigonometric ratios?* (Sample responses: S=o/h C=a/h T=o/a, soh cah toa, sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, tangent equals opposite over adjacent)

Ask: *What do you mean by opposite and adjacent?* (Sample responses: across from that thing, next to theta, the legs) Draw a right triangle and ask students to label the sides, but don't have theta there. Have them tell you they need theta and that the side names are relational to where theta is located. You can do this by asking how they know it is opposite/adjacent. It's opposite/adjacent to what? How do they know that angle?

Ask: *What was special about the sine and cosine ratios?* (The sine of an angle is equal to the cosine of its complementary angle.)

Ask: *How can you apply the trigonometric ratios to real life?* (Sample responses: to cross bridges, to find the length of a side, to find the measure of an angle)

Ask: *How else are triangles and geometric principles applied in real life?* (Sample responses: They are used to build bridges; they are used to build houses; they're in clothing design.)

- Bold text in the Instructional Sequence highlights explicit instruction to the Common Core standards.